Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

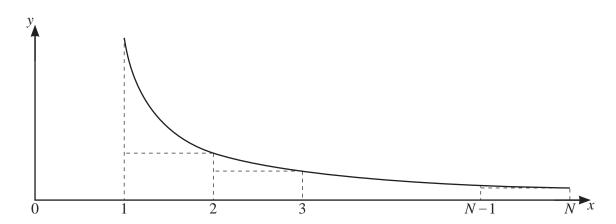
This document has 16 pages.

1	(a)	Find a and b such that	
		$z^8 - iz^5 - z^3 + i = (z^5 - a)(z^3 - b).$	[1]
	(b)	Hence find the roots of	
		$z^8 - iz^5 - z^3 + i = 0,$	
		giving your answers in the form $re^{i\theta}$, where $r > 0$ and $0 \le \theta < 2\pi$.	[6]

Find the Maclaurin's series for $\ln \cosh x$ up to and including the term in x^4 .	
	•••••

4

3



The diagram shows the curve $y = \frac{x}{2x^2 - 1}$ for $x \ge 1$, together with a set of N - 1 rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$\sum_{r=1}^{N} \frac{r}{2r^2 - 1} < \frac{1}{4} \ln r$	$n(2N^2 - 1) + 1.$	[7]

(b)	Use a similar method to find, in terms of N , a lower bound for $\sum_{r=1}^{N} \frac{r}{2r^2 - 1}$. [3]

4	By considering the binomial expansions Moivre's theorem to show that	of	$\left(z + \frac{1}{z}\right)^5$	and	$\left(z - \frac{1}{z}\right)^5$, where	$z = \cos\theta + \mathrm{i}\sin\theta,$	use de

$$\tan^5\theta = \frac{\sin 5\theta - a\sin 3\theta + b\sin \theta}{\cos 5\theta + a\cos 3\theta + b\cos \theta},$$

where a and b are integers to be determined.	[7]

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5	I ne	variables	s x ana y	are	related	by	tne	differential	equation

by the differential equation
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 4e^{-x}.$$

a)	Find the value of the constant k such that $y = kxe^{-x}$ is a particular integral of the different equation.	erentia [4
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b)	Find the solution of the differential equation for which $y = \frac{dy}{dx} = \frac{1}{2}$ when $x = 0$.	[6
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	$2\sinh^2 x = \cosh 2x - 1.$	
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b)	Find the solution to the differential equation dy	•••••
b)	Find the solution to the differential equation $\frac{dy}{dx} + y \coth x = 4 \sinh x$	
b)		•
b)	$\frac{\mathrm{d}y}{\mathrm{d}x} + y \coth x = 4 \sinh x$	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} + y \coth x = 4 \sinh x$	
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(b)	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$.	
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a)	Find the exact value of I_1 , expressing your answer in logarithmic form.	
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b)	By considering $\frac{d}{dx} \left(x \left(4 + x^2 \right)^{-\frac{1}{2}n} \right)$, or otherwise, show that $4nI_{n+2} = \frac{3}{2} \left(\frac{2}{5} \right)^n + (n-1)I_n.$	
b)	By considering $\frac{d}{dx}(x(4+x^2)^{-\frac{1}{2}n})$, or otherwise, show that $4nI_{n+2} = \frac{3}{2}(\frac{2}{5})^n + (n-1)I_n.$	
b)		
b)		
b)		

(c)	Find the value of I_5 . [3]

(a)	Find the value of a for which the system of equations	
	13x + 18y - 28z = 0,	
	-4x - ay + 8z = 0, 2x + 6y - 5z = 0,	
	does not have a unique solution.	[2]
	does not have a unique solution.	[4]
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		•••••
The	matrix \mathbf{A} is given by	••••
	$\mathbf{A} = \begin{pmatrix} 13 & 18 & -28 \\ -4 & -1 & 8 \\ 2 & 6 & -5 \end{pmatrix}.$	
	$\begin{pmatrix} 1 & 1 & 0 \\ 2 & 6 & -5 \end{pmatrix}$	
	$\begin{pmatrix} 2 \\ 2 \end{pmatrix}$	
(b)	Find the eigenvalue of A corresponding to the eigenvector $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$.	[1]
(c)	Find a matrix P and a diagonal matrix D such that $\mathbf{A} = \mathbf{PDP}^{-1}$.	[8]
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Use the characteristic equation of \mathbf{A} to find \mathbf{A}^{-1} in terms of \mathbf{A} .

(d)

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.					

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